International Journal of Engineering Research & Management Technology
ISSN: 2348-4039
March-2016 Volume 3, Issue-2
Email:editor@ijermt.org
www.ijermt.org

Propagation of Rayleigh waves in orthotropic prestressed solid with impedance boundary conditions

Amit Kumar
Research Scholar
Department of Mathematics
Singhania University, Rajasthan
amitgju.hisar@gmail.com

Inder Singh Gupta
Research Supervisor
Department of Mathematics
J.V.M.G.R.R. (P.G.) College, Charkhi Dadri, Bhiwani
is_gupta@yahoo.com

Abstract

The explicit secular equations for Rayleigh waves are important for evaluating the dependence of the wave velocity on material parameters and to determine material parameters from measured values of wave velocity. Usually, in the case of Rayleigh waves, it is always used that the half-spaces are free of traction. But in the field of physics particularly acoustic and electromagnetism, it is common to use impedance boundary conditions. We have derived the secular equation of Rayleigh waves propagating in an orthotropic prestressed elastic half-space with surface is subjected to the impedance boundary conditions.

Introduction

The explicit secular equations for Rayleigh waves are important for evaluating the dependence of the wave velocity on material parameters and to determine material parameters from measured values of wave velocity. Usually, in the case of Rayleigh waves, it is always used that the half-spaces are free of traction. But in the field of physics particularly acoustic and electromagnetism, it is common to use impedance boundary conditions. It is a linear combination of the unknown function and their derivatives on the boundary. Many researchers have been studying, the effect of thin layer on the half-space by the effective boundary conditions like Achenbach and Keshava (1967), Steigmann and Ogden (2007) and Vinh *et al.* (2014).

Vinh and Hue (2014) derived explicit secular equations of Rayleigh waves in orthotropic half-space using the traditional method. This equation coincides with the secular equation of Rayleigh waves with the secular equation of Rayleigh waves with traction free boundary conditions.

In this chapter, the propagation of Rayleigh waves in anisotropic elastic prestressed half-space subjected impedance boundary conditions is investigated. For orthotropic case, the secular equation is derived by using simple method. From this equation, a new secular equation of the wave is obtained for the isotropic case.

Basic Equations

Consider a prestressed medium which occupies the domain $y \geq 0$. The material is either isotropic in finite strain or anisotropic with orthotropic symmetry. The principal directions of prestress are choosen to coincide with the direction of elastic symmetry and the co-ordinate axes. Here we are interested in the plane strain such that $u = u(x,y,t)$, $v = v(x,y,t)$, $w = 0$ where t is time. The state of initial stress is therefore defined by principal components S_{11}, $S_{22} = S_{33}$. It is also assumed that S_{11} and S_{22} are constant. The general equations of motion for prestressed solid in the absence of external forces is given by Biot (1965).

$$s_{ij,j} + S_{jk}\omega_{ik,j} + S_{ik}\omega_{jk,j} - e_{jk}S_{ik,j} = \rho u_{i,tt}, \tag{1}$$

where ρ is the density, u_i are the displacement components and, i indicates differentiation with respect to x_i. The incremental stresses s_{ij} are assumed to be linearly related to the incremental strains e_{ij} through the incremental elastic coefficients B_{ij}, Q_1 and Q_2.

$$s_{11} = B_{11}u_x + B_{12}\left(v_y + w_z\right),$$

$$s_{22} = \left(B_{12} - P\right)u_x + B_{22}v_y + B_{23}w_z,$$

$$s_{33} = \left(B_{12} - P\right)u_x + B_{23}v_y + B_{22}w_z, \qquad (2)$$

$$s_{12} = Q_2\left(u_y + v_x\right),$$

$$s_{13} = Q_2\left(u_z + w_x\right),$$

$$s_{23} = Q_1\left(w_y + v_z\right).$$

where, it is assume that $S_{22} = S_{33}$, S_{11} and S_{33} are constants.

$$(x, y, z) = \left(x_1, x_2, x_3\right),$$

$$(u, v, w) = \left(u_1, u_2, u_3\right),$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \qquad (3)$$

$$\omega_{ij} = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right),$$

$$u_{i,j} = \frac{\partial u_i}{\partial x_j},\ u_{i,t} = \frac{\partial u_i}{\partial t},\ u_x = \frac{\partial u}{\partial x},\ \text{etc.},$$

$$P = S_{33} - S_{11},$$

and

$$B_{11} = (2\mu + \lambda)\left(1 + \varepsilon_{11} - 2\varepsilon_{22}\right),$$

$$B_{22} = (2\mu + \lambda)\left(1 - \varepsilon_{11}\right), \qquad (4)$$

$$B_{12} = \lambda\left(1 - \varepsilon_{22}\right) - S_{11},$$

$$B_{22} = \lambda\left(1 - \varepsilon_{11}\right) - S_{33},$$

$$Q_1 = \mu(\mu+\lambda)\varepsilon_{22} + \frac{1}{2}(\lambda - 2\mu)\varepsilon_{11},$$

$$Q_2 = \mu + \frac{1}{2}(\mu+\lambda)(\varepsilon_{11}+\varepsilon_{22}) + \frac{1}{2}(\lambda-2\mu)\varepsilon_{22}.$$

Further λ, μ are Lame's constant and ε_{ij} are the incremental strains, s_{ij} and ε_{ij} are related by Hooke's Law.

$$s_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{5}$$

Here it is assumed that material properties and initial stress components are varying as

$$\lambda = \lambda^0 e^{az},\ \mu = \mu^0 e^{az},\ \rho = \rho^0 e^{az} \text{ and } S_{11} = S_{11}^0 e^{az},\ S_{22} = S_{22}^0 e^{az},$$

$$S_{33} = S_{33}^0 e^{az}, P = P^0 e^{az}\,(a > 0). \tag{6}$$

Then the incremental elastic coefficients becomes

$$B_{11} = B_{11}^0 e^{az},\ B_{22} = B_{22}^0 e^{az},\ B_{12} = B_{12}^0 e^{az},\ B_{23} = B_{23}^0 e^{az} \tag{7}$$

$$Q_1 = Q_1^0 e^{az},\ Q_2 = Q_2^0 e^{az},\ P^0 = S_{33}^0 - S_{11}^0.$$

where $\left(B_{11}^0, B_{22}^0, B_{12}^0, B_{23}^0, Q_1^0, Q_2^0\right)$ and $\left(S_{11}^0, S_{33}^0\right)$ are elastic coefficients and initial stresses in homogeneous orthotropic prestressed medium. λ^0, μ^0, ρ^0 are Lame's constants and density of material at the free surface.

Using Eqs. (2), (3), (6) and (7) in Eq.(1), we get

$$B_{11}^0\frac{\partial^2 u}{\partial x^2} + \left(B_{12}^0 + Q_2^0 - \frac{P^0}{2}\right)\frac{\partial^2 w}{\partial x\partial z} + \left(B_{12}^0 + Q_2^0 - \frac{P^0}{2}\right)\frac{\partial^2 v}{\partial x\partial y} + \left(Q_2^0 + \frac{P^0}{2}\right)\frac{\partial^2 u}{\partial y^2}$$

$$+\left(Q_2^0 + \frac{P^0}{2}\right)\frac{\partial^2 u}{\partial z^2} + a\left(Q_2^0 + \frac{P^0}{2}\right)\frac{\partial u}{\partial z} + a\left(Q_2^0 - \frac{P^0}{2}\right)\frac{\partial w}{\partial x} = \rho^0\frac{\partial^2 u}{\partial t^2},$$

$$\left(B_{12}^0 + Q_2^0 - \frac{P^0}{2}\right)\frac{\partial^2 u}{\partial x\partial y} + \left(Q_2^0 - \frac{P^0}{2}\right)\frac{\partial^2 v}{\partial x^2} + B_{22}^0\frac{\partial^2 v}{\partial y^2}$$

$$\left(B_{23}^0 + Q_1^0\right)\frac{\partial^2 w}{\partial y\partial z} + Q_1^0\frac{\partial^2 v}{\partial z^2} + aQ_1^0\frac{\partial w}{\partial y} + aQ_1^0\frac{\partial v}{\partial z} = \rho^0\frac{\partial^2 v}{\partial t^2}, \tag{8}$$

$$\left(Q_2^0 - \frac{P^0}{2}\right)\frac{\partial^2 w}{\partial x^2} + \left(B_{12}^0 + Q_2^0 - \frac{P^0}{2}\right)\frac{\partial^2 u}{\partial x \partial z} + Q_1^0 \frac{\partial^2 w}{\partial y^2} + \left(B_{23}^0 + Q_1^0\right)\frac{\partial^2 v}{\partial y \partial z}$$

$$+B_{22}^0 \frac{\partial^2 w}{\partial z^2} + a\left(B_{12}^0 - P^0\right)\frac{\partial u}{\partial x} + aB_{23}^0 \frac{\partial v}{\partial y} + aB_{22}^0 \frac{\partial w}{\partial z} = \rho^0 \frac{\partial^2 w}{\partial t^2}.$$

The incremental elastic constants satisfy the inequalities.

$$Q > 0,\ B_{ii} > 0,\ i = 1,2 \ \text{ and } \ B_{11}B_{23} - B_{12}^2 > 0 \tag{9}$$

which are necessary and sufficient conditions for the strain energy to be positive definite. In the absence of body force, the equations of motion for incremental deformation are given as (Eq. 8) putting a = 0;

$$B_{11}\frac{\partial^2 u}{\partial x^2} + A_3 \frac{\partial^2 v}{\partial x \partial y} + A_1 \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2},$$

$$B_{22}\frac{\partial^2 v}{\partial y^2} + A_3 \frac{\partial^2 u}{\partial x \partial y} + A_1 \frac{\partial^2 v}{\partial x^2} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{10}$$

where,

$$A_1 = Q^0 + \frac{P^0}{2},\ A_2 = Q^0 - \frac{P^0}{2}$$

$$A_3 = B_{12}^0 + Q^0 - \frac{P^0}{2}. \tag{11}$$

If the medium is isotropic elastic homogeneous solid and the first order theory of classical elasticity is assumed then it may be shown that (Biot, pp. 111-112)

$$B_{11}^0 = B_{22}^0 = \lambda^0 + 2\mu^0,\ B_{12}^0 = \lambda^0,\ Q^0 = \mu,\ A_1 = A_2 = \mu^0,\ A_3 = \lambda^0 + \mu^0 \tag{12}$$

We consider the propagation of a Rayleigh wave travelling with velocity $C_R(>0)$ and wave number $K(>0)$ in the *x*-direction and decaying in the *y*-direction, i.e.

$$u \text{ and } v \to 0 \ \text{ as } \ y \to \infty \tag{13}$$

Boundary Conditions

We assume, the surface y = 0 is subjected to impedance boundary conditions such that

$$\Delta f_x + \omega z_1 u = 0,\ \Delta f_y + z_2 \omega z_2 v = 0 \text{ at } y = 0, \tag{14}$$

where, Δf_x and Δf_y are incremental forces given by (Biot, p-4) as

$$\Delta f_x = s_{12} - S_{22}\omega_z - S_{11}e_{xy},$$

$$\Delta f_y = s_{22} + S_{22}e_{xx}, \tag{15}$$

where, e_{xy}, e_{xx} are incremental strains and ω_x, ω_z are the incremental rotation component parallel to xy-plane. Explicit expressions for these quantities in terms of u and v are (Biot p-321)

$$e_{xx} = \frac{\partial u}{\partial x},\ e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),\ \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{16}$$

and $\omega = kc$ is the wave circular frequency, $z_1, z_2 (\in R)$ is impedance parameters whose dimension is of stress per velocity.

Let the solution of Eqs. (10) in displacements form as

$$u = U_r e^{-ik(C_R t - x) - qky},$$

$$v = V_r e^{-ik(C_R t - x) - qky} \tag{17}$$

where, U_r and V_r are the amplitude factors and q is assumed to be real and positive. U_r and V_r are functions of y also. Using Eq. (17) in Eq. (10), we get

$$\left(\rho C_R^2 - B_{11} + A_1 q^2\right)U_r + iqA_3 W_r = 0$$

$$iqA_3 U_r + \left(\rho C_R^2 + q^2 B_{22} - A_2\right)W_r = 0 \tag{18}$$

On simplification, we get

$$q^4 B_{22} A_1 + \left\{A_3^2 + C_{22}\left(\rho C_R^2 - B_{11}\right) + A_1\left(\rho C_R^2 - A_2\right)\right\}q^2$$

$$+\left(C_{11} - \rho C_R^2\right)\left(C_{66} - \rho C_R^2\right) = 0 \tag{19}$$

It is quadratic equation in q^2. Let q_1^2 and q_2^2 are two values of Eq. (19) and

$$q_1^2 + q_2^2 = -\frac{\left\{A_3^2 + C_{22}\left(\rho C_R^2 - B_{11}\right) + A_1\left(\rho C_4^2 - A_2\right)\right\}}{B_{22}A_1} = S_1 . \tag{20}$$

and

$$q_1^2 q_2^2 = \frac{\left(C_{11} - \rho C_R^2\right)\left(C_{66} - \rho C_R^2\right)}{B_{22}A_1} = P_1 .$$

From first and second member of Eq. (18), we get

$$\frac{V_{rk}}{U_{rk}} = i\left(\frac{q_1 A_3}{\rho C_R^2 + q_i^2 B_{22} - A_2}\right) = i\left(\frac{B_{11} - \rho C_R^2 - A_1 q_i^2}{q_i A_3}\right) = im_k = M_k \tag{21}$$

where

$$m_k = \frac{q_k A_3}{\rho C_R^2 + q_k^2 B_{22} - A_2} = \frac{B_{11} - \rho C_R^2 - A_1 q_k^2}{q_k A_3}, k = 1, 2, \ldots$$

Then Eq. (17) becomes

$$u = \left\{U_{r_1} e^{-q_1 ky} + U_{r_2} e^{-q_2 ky}\right\} e^{ik(x - C_R t)}$$

$$\upsilon = \left\{M_1 U_{r_1} e^{-q_1 ky} + M_2 U_{r_2} e^{-q_2 ky}\right\} e^{ik(x - C_R t)} \tag{A}$$

It is not difficult to verify from second member of Eq. (20) that if a Rayleigh wave exist then

$$0 < \rho C_R^2 < \min\left(A_1, B_{22}\right) \tag{22}$$

and

$$q_1 q_2 = \sqrt{P_1},\ q_1 + q_2 = \sqrt{S + 2\sqrt{P_1}} \tag{23}$$

Substituting Eq. (A) into Eq. (15) yields

$$\left\{\left(Q_2 + \frac{P}{2}\right) q_1 + m_1\left(Q - \frac{R}{2}\right) - C_R z_1\right\} U_{r_1}$$

$$+\left\{\left(Q + \frac{P}{2}\right) q_2 + m_2\left(Q - \frac{R}{2}\right) - C_R z_1\right\} U_{r_2} = 0$$

$$\left\{\left(B_{12} + S_{11}\right) - m_1 B_{22} q_1 + m_1 c z_2\right\} U_{r_2}$$

$$+\left\{\left(B_{12}+S_{11}\right)-m_2B_{22}q_2+m_2cz_2\right\}U_{r_2}=0 \qquad (24)$$

Since $U_{r_2}^2+U_{r_1}^2\neq 0$, the determinate of the system Eq. (24) must vanish. This yields

$$\begin{vmatrix}\left(Q+\frac{P}{2}\right)q_1+m_1\left(Q-\frac{R}{2}\right)-C_Rz_1 & \left(Q+\frac{P}{2}\right)q_2+m_2\left(Q-\frac{R}{2}\right)-C_Rz_1 \\ \left(B_{12}+S_{11}\right)-m_1q_1B_{22}+m_1C_Rz_2 & \left(B_{12}+S_{11}\right)-m_2q_2B_{22}+m_2z_2C_R\end{vmatrix}=0 \qquad (25)$$

or

$$\left\{\left(Q_2+\frac{P}{2}\right)q_1+m_1\left(Q-\frac{R}{2}\right)+\frac{\omega z_1}{k}\right\}\left\{i\left(B_{12}+S_{11}\right)-B_{22}m_2q_2+C_Rz_2m_2\right\}$$

$$-\left\{\left(Q+\frac{P}{2}\right)q_2+m_2\left(Q-\frac{R}{2}\right)+C_Rz_1\right\}\left\{\left(B_{12}+S_{11}\right)-m_1q_1B_{22}+m_1C_Rz_2\right\}=0$$

(26)

This is the secular equation of Rayleigh waves propagating in an orthotropic prestressed elastic half-space with surface is subjected to the impedance boundary conditions.

Particular Cases

Case 1: For unstressed homogeneous orthotropic elastic half-space with surface is subjected to the impedance boundary conditions.

Put $P=R=0\Rightarrow S_{11}=S_{33}=0$ in Eq. (26)

$$\left\{\left(q_1+m_1\right)-\frac{C_Rz_1}{Q}\right\}\left\{B_{12}-m_2q_2B_{22}+m_2z_2C_R\right\}$$

$$-\left\{\left(q_2+m_2\right)-\frac{C_Rz_1}{Q}\right\}\left\{B_{12}-m_1q_1B_{22}+m_1C_Rz_2\right\}=0 \qquad (27)$$

Put $x = \dfrac{C_R^2}{C_2^2}, C_2^2 = \dfrac{Q}{\rho}$ is squared dimensionless velocity of Rayleigh waves, $\delta_n = \dfrac{Z_n}{\sqrt{PQ}} \in R,\ n = 1,2$ are dimensionless impedance parameters. Then Eqs. (27) becomes

$$\left(\delta_1\sqrt{x} - q_1 - m_1\right)\left(\delta_2 m_2\sqrt{x} + B_{12} - m_2 q_2 B_{22}\right)$$

$$-\left(\delta_1\sqrt{x} - q_2 - m_2\right)\left(\delta_2 m_1\sqrt{x} + B_{12} - m_1 q_1 B_{22}\right) = 0 \qquad (28)$$

Put $e_1 = \dfrac{B_{11}}{Q}, e_2 = \dfrac{B_{22}}{Q}$ and $e_3 = \dfrac{B_{12}}{Q}$ are dimensionless material parameters

$$\left(\delta_1\sqrt{x} - q_1 - m_1\right)\left(\delta_2 m_2\sqrt{x} + e_3 - m_2 q_2 e_2\right)$$

$$-\left(\delta_1\sqrt{x} - q_2 - m_2\right)\left(\delta_2 m_1\sqrt{x} + e_3 - m_1 q_1 e_2\right) = 0$$

On simplification, we get

$$\left(\delta_1\delta_2 x + e_3 + e_2 q_1 q_2\right)\left(m_2 - m_1\right) + \left(e_3 + e_2 m_1 m_2\right)\left(q_2 - q_1\right)$$

$$-\delta_1 e_2\left(m_2 q_2 - m_1 q_1\right)\sqrt{x} - \delta_2\left(m_2 q_1 - m_1 q_2\right)\sqrt{x} = 0 \qquad (29)$$

That is the dimensionless equation of Rayleigh waves derived by Vinh and Hue (2014). Dimensionless material parameters also satisfy the following condition like elastic constants as

$$e_1 > 0,\ e_2 > 0 \text{ and } e_1 e_2 - e_3^2 > 0. \qquad (30)$$

We can solve the following inequalities

$$m_2 - m_1 = -\frac{e_1 - x + q_1 q_2}{\left(e_3 + 1\right) q_1 q_2}\left(q_2 - q_1\right),$$

$$m_2 m_1 = \frac{e_1 - x}{e_2 q_1 q_2}, \qquad (31)$$

$$P_1 = \frac{\left(e_1 - x\right)\left(1 - x\right)}{e_2} \text{ and } S_1 = \frac{e_2\left(e_1 - x\right) + 1 - x - \left(1 + e_3\right)^2}{e_2}$$

Case II: Taking $\delta_1 = \delta_2 = 0$ in Eq. (29), we get

$$(q_1 + m_2)(B_{12} - m_2 q_2 B_{22}) - (q_2 + m_2)(B_{12} - m_1 q_1 B_{22}) = 0 \quad (32)$$

On simplification and using Eq. (31)

$$q_1 B_{12} + m_1 B_{12} - q_1 q_2 m_2 B_{22} - m_1 m_2 q_2 B_{22} - q_2 B_{12} - m_2 B_{12}$$

$$+ q_1 q_2 m_1 B_{22} + m_1 m_2 q_1 B_{22} = 0$$

$$(q_1 - q_2) B_{12} + (m_1 - m_2) B_{12} + q_1 q_2 (m_1 - m_2) B_{22} + m_1 m_2 (q_1 - q_2) B_{22} = 0$$

$$(q_1 - q_2)(B_{12} + m_1 m_2 B_{22}) + (B_{12} + q_1 q_2 B_{22})(m_1 - m_2) = 0$$

$$(q_2 - q_1)(B_{12} + m_1 m_2 B_{22}) - (B_{12} + q_1 q_2 B_{22})\left(\frac{e_1 - x + q_1 q_2}{(e_3 + 1) q_1 q_2}\right)(q_2 - q_1) = 0$$

$$(B_{12} + m_1 m_2 B_{22}) - \frac{(B_{12} + q_1 q_2 B_{22})(e_1 - x + q_1 q_2)}{(e_3 + 1) q_1 q_2} = 0$$

$$(e_3 + 1) q_1 q_2 \left(B_{12} + \frac{(e_1 - x) e_2}{e_2 q_1 q_2}\right) - (B_{12} + q_1 q_2 B_{22})(e_1 - x + q_1 q_2) = 0$$

$$(e_3 + 1)(e_2 q_1 q_2 B_{12} + e_1 - x) - e_2 (e_3 + q_1 q_2 e_2)(e_1 - x + q_1 q_2) = 0$$

$$(e_3 + 1)(e_2 e_3 q_1 q_2 + e_2 (e_1 - x)) - e_2 (e_3 + q_1 q_2 e_2)(e_1 - x - q_1 q_2) = 0$$

$$\{e_2 e_3 (e_3 + 1) - e_2 e_3 - e_2^2 (e_1 - x)\} q_1 q_2 + (e_3 + 1)(e_1 - x) e_2$$

$$- e_2 e_3 (e_1 - x) - e_2^2 q_1^2 q_2^2 = 0$$

$$\{e_2 e_3^2 + e_2 e_3 - e_2 e_3 - e_2^2 (e_1 - x)\} \sqrt{P} + e_2 (e_1 - x)(e_3 + 1 - e_2 e_3) - e_2^2 P = 0$$

$$e_2 (e_3^2 - e_1 e_2 + e_2 x) \sqrt{\frac{(1 - x)(e_1 - x)}{e_2}} + (e_1 - x) e_2 - e_2 (1 - x)(e_1 - x) = 0$$

$$e_2 (e_3^2 - e_1 e_2 + e_2 x) \sqrt{\frac{(1 - x)(e_1 - x)}{e_2}} + e_2 (e_1 - x)(1 - (1 - x)) = 0$$

$$e_2 \left\{(e_3^2 - e_1 e_2 + e_2 x) \sqrt{\frac{(1 - x)(e_1 - x)}{e_2}} + (e_1 - x) x\right\} = 0$$

$$\left(e_3^2 - e_1 e_2 + e_2 x\right)\sqrt{\frac{(1-x)(e_1-x)}{e_2}} + (e_1 - x)x = 0 \tag{33}$$

It is equivalent to the secular equation given by Chandwick (1976).

Case III: When the elastic half-space is isotropic i.e.

$B_{11} = B_{22} = \lambda^0 + 2\mu^0$, $B_{12} = \lambda^0$ and $Q = \mu^0$, λ^0 and μ^0 are Lame's constants as given in Eq. (12).

From Eqs. (19) and (21), we get

$$m_1 = q_1,\ m_2 = \frac{1}{q_2}$$

where q_1 and q_2 are

$$q_1 = \sqrt{1-rx} \text{ and } q_2 = \sqrt{1-x}, \text{ where } r^0 = \frac{\mu^0}{\lambda^0 + 2\mu^0}$$

Putting $e_1 = e_2 = \frac{1}{r^0}$, $e_3 = \frac{1}{r^0} - 2$ and the values of $m_2 - m_1$ and $m_1 m_2$ from Eq. (31), we get

$$(x-2)^2 - 4\sqrt{1-x}\sqrt{1-rx} + x\sqrt{x}\left(\delta_1\sqrt{1-x} + \delta_2\sqrt{1-rx}\right)$$

$$+ x\delta_1\delta_2\left(\sqrt{1-x}\sqrt{1-rx} - 1\right) = 0 \tag{34}$$

It coincides with the results of Godo *et al.* (2012) and Malischewsky (1987).

Again the value of $\sqrt{S + 2\sqrt{P}} = q_1 + q_2 = \sqrt{1-rx} + \sqrt{1-x}$ for the isotropic elastic half-spaces. The Eq. (21) becomes

$$x\left(1-r^0x\right)\left(1-\delta_1\delta_2\right) + \left[x + 4\left(r^0 - 1\right) - r^0\delta_1\delta_2 x\right]\sqrt{1-x}\sqrt{1-r^0x}$$

$$-\left(\delta_1+\delta_2\right)\left(1-r^0x\right)\sqrt{x}\sqrt{1-x} - \left[\delta_1(1-x) + \delta_2\left(1-r^0x\right)\right]\sqrt{x}\sqrt{1-r^0x} = 0 \tag{35}$$

Eq. (35) is dimensionless secular equation of Rayleigh waves propagating in an isotropic elastic half-space subjected to the impedance boundary conditions.

Put $\delta_2 = 0$, Eq. (34), on simplification, we get

$$(x-2)^2 - 4\sqrt{1-x}\sqrt{1-r^0 x} + \delta_1 x\sqrt{x}\sqrt{1-x} = 0.$$

Conclusions

The secular equations of Rayleigh waves in prestressed orthotropic half-space are derived under impedance boundary conditions. The secular equations obtained containing impedance parameters and initial stresses also. The secular equation of the Rayleigh waves for the isotropic case is deduced from prestressed orthotropic media that is identical to Eq. (9) (Godoy *et al.*, 2012).

References

[1] A.Ben-Menahem and S.J. Singh, Seismic waves and sources,Springer-verlag,New York,(1981).

[2] A.M. Abd-Alla,S.R. Mahmoud and M.I.R. Helmi,Effect of initial stress and magnetic field on propagation of shear wave in non-homogeneousanisotropic medium under gravity field, Open Appl. Math. J, 3, 49-56,(2009).

[3] A.N. Norries, Propagation of plane waves in a prestressed elastic medium ,J.Acoust. Soc. Am., 74,1642-1643,(1983).

[4] J.T. Wilson , Surface waves in a heterogeneous media, Bull. Seism. Soc. Am. ,32,297-304,(1942).

[5] J.F. Hook, Separation of the vector Wave equation of elasticity for certain types of inhomogeneous isotropic media, J.Acoust. Soc. Am.,33(3),302-313,(1961).

[6] K. Singh ,Propagation of Rayleigh waves in an axially symmetric heterogeneous layer lying between two homogeneous half space, pure and appl. Geophysics.61(1),36-44,(1965).

[7] L.M. Brekhovskikh , Waves in layered media, academic press, New York,(1960).

[8] M.A. Biot, Mechanics of incremental deformation , john Wiley and sons,New York,(1965).

[9] M.Newlands , Rayleigh waves in a two layer heterogeneous medium, Monthly notice of Roy.Astron.Soc., geophysical Suppl. 6(2) ,109-124,(1950).

[10] R. Stoneley , The transmission of Rayleigh waves in a heterogeneous medium , monthly notices of the Roy Astron. Soc.,Geophysical Suppl. 3(6),222-232,(1934).

[11] R.S. Sidhu , Transmission of Rayleigh waves over the surface of a heterogeneous medium, Pure and appl. Geophysics ,80(1),48-70,(1970).

[12] R.S. Sidhu and S.J. Singh ,Comments on reflection elastic waves under initial stress at a free surface: P and SV motion ,J.Acoust.Soc.Am.,74,1940-1944,(1983).

[13] R. Kakar and S. Kakar ,Rayleigh waves in a non-homogeneous granular media,"J. Chem. Bio. Phy. Sci. Sec. B, 3(1), 464-478,(2013).

[14] S.C. Das , D.P. Acharya and P.R. Sengupta , Surface waves in an inhomogeneous elastic medium under the influence of gravity, Rev. Roum. Des. Sci. Tech., **37**, 539–551,(1992).

[15] S. Dutta, Rayleigh waves in two layer heterogeneous medium , Bull. Seism. Soc. Am. 53(5), 1007-1022, (1963).

[16] W.L. Pilant , Elastic Waves in the Earth, Elsevier sci. publ. Comp. Amsterdam,(1979).

[17] Vinh, P.C. and Hue, T.T.T., "Rayleigh waves with impedance boundary conditions in anisotropic solids," Wave Motion, 51 (2014), 1082-1092.